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Structure of Phenomenological Lagrangians for Broken Supersymmetry

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ABSTRACT

We consider the explicit connection between linear representations of supersymmetry and the nonlinear realizations associated with the generic effective Lagrangians of the Volkov-Akulov type. We specify and illustrate a systematic approach for deriving the appropriate phenomenological Lagrangian by transforming a pedagogical linear model, in which supersymmetry is broken at the tree level, into its corresponding nonlinear Lagrangian, in close analogy to the linear σ model of pion dynamics. We discuss the significance and some properties of such phenomenological Lagrangians.

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I. INTRODUCTION

A realistic theory employing supersymmetry principles should ultimately include a satisfactory treatment of the breaking of supersymmetry. A great deal of work has been devoted to the study of models for supersymmetry breaking,[1,2] the super-Nambu-Goldstone mechanism,[2] low energy theorems,[3] and considerations of dynamical schemes.[4] Moreover, the universal nonlinear realizations characteristic of broken supersymmetry and their Lagrangians were constructed early on in the development of the theory.[5,6] Specifically, Volkov and Akulov[5] have introduced the nonlinear supersymmetry transformation laws and Lagrangians involving the massless Goldstone fermion associated with the process, as well as its general coupling to spectator fields (which do not play a Nambu-Goldstone role in the theory) of arbitrary spin and multiplicity.

Ivanov and Kapustnikov[7] have extended this construction by providing transformation laws which connect linear representations with the nonlinear realizations of supersymmetry, in the framework of the classic general discussion of coset realizations by Coleman, Wess and Zumino.[8] (Since, in most respects, the concepts involved were introduced a dozen years ago in the context of chiral dynamics,[8,9] we will be outlining direct analogies to soft pion physics and the sigma model[10] where appropriate.)

Roček[11] has provided examples of constraining the fields of linear theories to eliminate all spectator fields and then reexpressing these systems into that Volkov-Akulov Lagrangian which involves only the Goldstone spinor--this corresponds to eliminating the σ field in the nonlinear sigma model to derive the nonlinear π Lagrangian.

Nonetheless, it should be interesting to retain <u>all</u> of the original degrees of freedom in a given linear, renormalizable theory and to fashion it into its equivalent nonlinear, nonmanifestly renormalizable, effective Lagrangian of the Volkov-Akulov type, by pruning out <u>only</u> the superfluous fields present. This effective Lagrangian should be equivalent to the original one, and, in the framework of broken supersymmetry, it should produce the same tree level S-matrix amplitudes. In this paper we provide computational status to this program and illustrate our procedure on a simple prototype model which retains all the essential features of the general case.

Phenomenological Lagrangians isolate the Goldstone degrees of freedom in a system with broken symmetry[9,8] and generally provide a heuristic guide to its low energy behavior. For instance, in the case of chiral symmetry breaking, effective Lagrangians summarize the low energy structure derivable from current algebra. An example of their relevance in broken supersymmetry is the interesting suggestion[12] that, if quarks and leptons were to be considered as composite objects, they could be protected from acquiring too high a mass by the Nambu-Goldstone mechanism. Specifically, the observable fermions could be thought of as pseudogoldstone fermions of dynamically broken supersymmetry, taking their relatively small mass from a weak explicit breaking of supersymmetry associated with the low energy gauge interactions--an analogue of the pseudogoldstone feature of the pion in chiral symmetry breaking. At very low energies the supersymmetric interactions would be invisible because of Adler decoupling. The intermediate energy phenomenology of this model may be studied conveniently through the suitable effective Lagrangian of the Volkov-Akulov type, even in the absence of the equivalent linear theories.

Our paper is organized as follows. We first review, discuss, and clarify some known general features of supersymmetric phenomenological Lagrangians (Section II). In Section III, we describe how to formally convert a linear model in which supersymmetry is broken at the tree level to its equivalent nonlinear Volkov-Akulov Lagrangian, by tuning out any superfluous fields involved. We explicitly illustrate the procedure by a two dimensional (super Φ^3) scalar multiplet; we wish to stress however that no essential complications are expected to arise in four dimensions, save the proliferation of fields and a marked increase in tedium (as dramatically exemplified in Ref. [11], which works out the same model in both two and four dimensions). In contrast to the four dimensional case,[1,2] two dimensions afford a linear model with a stable tree-level Nambu-Goldstone solution, which only contains a real scalar field and a Majorana spinor, and succinctly displays the features of semiclassical supersymmetry breaking with a minimum of complication. In principle, the procedure is valid for an arbitrary number of dimensions and size of multiplet.

In the next section (IV), we compute a few tree amplitudes for both the linear and the nonlinear theories to verify that they coincide, in accord with the phenomenological Lagrangian equivalence relation:[8] the tree approximation to the S-matrix is identical for both (nonlinearly) equivalent theories. The amplitudes evince Adler decoupling of the Goldstone particle at low energies, as dictated by the supercurrent algebra low energy theorems,[3] which do not have to depend directly on a particular Lagrangian realization. For the purpose of illustration, we work out such a simple theorem for spinor-scalar scattering (analogous to the Adler-Weisberger relation for πN scattering) and thereby reproduce the low energy behavior of the amplitude computed from the Lagrangian.

In Sect. V we briefly summarize our procedure and discuss its relevance to the breaking of supersymmetry. We conclude by illustrating how to apply the procedure to extended supersymmetry and constrained superfields.

Some conventions, formal manipulations, and technical details are elaborated in the Appendix.

Throughout the paper we will be using Majorana spinors for formal convenience, but conversion to Dirac spinors should present no difficulty: $\psi_{Dirac}=(\psi_{Maj}+i\psi'_{Maj})/\sqrt{2}$. Indices are suppressed when obvious.

II. GENERAL PROPERTIES OF THE NONLINEAR SUPERSYMMETRIC LAGRANGIANS

The Volkov-Akulov nonlinear realization of the supersymmetry albegra contains one massless spinor λ^a, which transforms inhomogeneously and nonlinearly under a spinorial variation of constant parameter ε:

$$\delta\lambda^a = f\varepsilon^a - \frac{i}{f}\,\partial_\mu\lambda^a\;\bar{\varepsilon}\gamma^\mu\lambda \tag{2.1}$$

f is a constant of dimension 2 in 4 spacetime dimensions (and dimension 1 in two dimensions) which parameterizes the breaking of supersymmetry: $\langle\delta\lambda^a\rangle = f\varepsilon^a$ --it is analogous to the pion decay constant f_π. It can be supressed and then reinstated, at any subsequent stage of our discussion, by dimensional analysis.

Of course, λ is a linear representation of the unbroken Poincaré subgroup of the full supersymmetry group. No other fields are needed[8] to make the realization faithful. The commutator of two infinitesimal variations (2.1) is the translation dictated by the supersymmetry algebra:

$$[\delta',\delta]\;\lambda^a = 2i\;\bar{\varepsilon}'\gamma_\mu\varepsilon\;\partial^\mu\lambda^a\;. \tag{2.2}$$

In order to find a supersymmetric Lagrangian for this fermion, we construct invariants, up to a divergence, under variations (2.1). Motivated by an underlying superspace transformation considered below (2.18), it is useful to define the 4×4 matrix:

$$W_\mu \equiv \delta_\mu^\nu + T_\mu{}^\nu, \tag{2.3}$$

where $$T_\mu{}^\nu \equiv \frac{-i}{f^2}\,\bar{\lambda}\,\gamma_\mu \partial^\nu \lambda \ , \tag{2.4}$$

and $$W_\mu^{-1\nu} = \delta_\mu^\nu - T_\mu{}^\nu + T_\mu{}^\kappa T_\kappa{}^\nu + \ldots \tag{2.5}$$

The series in T terminates with T^4, since $\lambda(\bar{\lambda}\lambda)^2=0$. The determinant of the 4×4 matrix W_μ^ν transforms as a total divergence under (2.1).

$$|W| \equiv \det W_\mu{}^\nu \equiv 1 + T_\mu^\mu + \frac{1}{2}\,(T_\mu^\mu\, T_\nu^\nu - T_\mu{}^\nu T_\nu{}^\mu) + 0(T^3) + 0(T^4). \tag{2.6}$$

To see this, observe that:

$$\delta W_\mu^\nu = \xi_\kappa\, \partial^\kappa W_\mu{}^\nu + \partial^\nu \xi_\kappa\, W_\mu{}^\kappa \tag{2.7}$$

where $$\xi_\kappa \equiv \frac{-i}{f}\,\bar{\varepsilon}\gamma_\kappa \lambda \ , \tag{2.8}$$

and therefore

$$\delta|W| = |W|\ W_{\nu}^{-1\mu}\ \delta W_{\mu}{}^{\nu} =$$

$$= |W|\ W_{\nu}^{-1\mu}(\xi_{\kappa}\partial^{\kappa}W_{\mu}{}^{\nu} + W_{\mu}\partial^{\nu}\xi_{\kappa})$$

$$= \partial^{\kappa}(\xi_{\kappa}|W|)\ . \qquad (2.9)$$

The simplest supersymmetric Lagrangian describes a massless fermion self-interacting through 4, 6, and 8-Fermi terms:

$$= -\frac{f^2}{2}\ |W| = -\frac{f^2}{2} + \frac{i}{2}\ \bar{\lambda}\not{\partial}\lambda + \ldots + 0(f^{-6})\ . \qquad (2.10)$$

Note the positive vacuum energy $f^2/2$ which also signals supersymmetry breaking.[4] The supercurrent is easily derived from (2.10) by Noether's procedure:

$$S_{\mu} = i\ f\ \gamma_{\nu}\lambda\ W_{\mu}^{-1\nu}|W| = i\ f\ \gamma_{\mu}\lambda\ + \ldots \qquad (2.11)$$

and, by its definition, it is conserved through the equations of motion:

$$2\ \partial_{\mu}\lambda\ W^{-1\mu}{}_{\nu}|W| + \lambda\ \partial_{\mu}(W^{-1\mu}{}_{\nu}|W|) = 0\ . \qquad (2.12)$$

The supercurrent couples the Goldstone spinor to the vacuum with strength f:[2,4]

$$\langle 0|S_{\mu}^{a}|\lambda^{b}\rangle = i\ f\ \gamma_{\mu}^{ab}\ . \qquad (2.13)$$

Also, integrating (2.11), one may verify through anticommutation that the vacuum energy is indeed $f^2/2$, as it should[4]--a fact which could be obscured if the constant term in the Lagrangian (2.10) were carelessly dropped.

We shall call fields of arbitrary spin not involved in a Nambu-Goldstone role spectators and denote them generically by ρ. (They correspond to σ or, in general, to "radial" degrees of freedom in chiral dynamics.) Their transformation law is a homogeneous, (λ-dependent) translation by ξ:

$$\delta\rho = -\frac{i}{f}\,\partial_\mu\rho\;\bar{\varepsilon}\gamma^\mu\lambda\ . \tag{2.14}$$

These transformations also satisfy the algebra (3.2).

Any local function of spectators transforms as a divergence, provided that it is weighted by $|W|$:

$$\begin{aligned}\delta(F(\rho)|W|) &= \partial_\kappa(\xi^\kappa|W|)F(\rho) + |W|\ \xi^\kappa\partial_\kappa F(\rho)\\ &= \partial_\kappa(\xi^\kappa|W|F(\rho))\ .\end{aligned} \tag{2.15}$$

Derivatives of either λ or spectators do not transform simply. However, the following covariant derivative may be defined for convenience:

$$\nabla^\mu \equiv W^{-1\mu}_{\nu}\partial^\nu\ . \tag{2.16}$$

Then it is straightforward to verify that covariant derivatives of both λ and ρ transform like spectators:

$$\delta(\nabla^{\mu}\rho) = W_{\nu}^{-1\mu}\partial^{\nu}(\xi^{\kappa}\partial_{\kappa}\rho) + \partial^{\nu}\rho(\xi^{\kappa}\partial_{\kappa}W_{\nu}^{-1\mu} - W_{\kappa}^{-1\mu}\partial^{\kappa}\xi^{\nu})$$

$$= \xi^{\kappa}\,\partial_{\kappa}(\nabla^{\mu}\rho) \tag{2.17}$$

Clearly, any Lorentz scalar function of ρ, $\nabla\rho$, and $\nabla\lambda$ multiplied by an overall $|W|$ may enter into a supersymmetrically invariant nonlinear Lagrangian: its particular form will depend on the features of the associated linear model, or on the supercurrent algebra to be realized.

The reader may have noted the formal analogy of $W_{\mu}{}^{\nu}$ with the Vierbein of General Relativity in the transformation properties, the covariant derivatives, and the construction of invariants. This is symptomatic of the origin of the above construction in a superspace translation[5,6] with a local parameter $-\lambda(x)/f$:

$$x'_{\mu} = x_{\mu} - \frac{i}{f}\,\bar{\lambda}(x)\gamma_{\mu}\theta$$

$$\theta' = \theta - \frac{\lambda(x)}{f}\,. \tag{2.18}$$

The Jacobian of this transformation, $J(x,\theta,\lambda(x)) = \partial(x',\theta')/\partial(x,\theta)$, is an 8×8 matrix which transforms the covariant vectors:

$$J_{\nu}^{b}{}_{a'}^{\mu'} \begin{pmatrix} \partial'_{\mu} \\ \\ \dfrac{\partial}{\partial \bar{\theta}^{a'}} \end{pmatrix} = \begin{pmatrix} \partial_{\nu} \\ \\ \dfrac{\partial}{\partial \bar{\theta}^{b}} \end{pmatrix} \tag{2.19}$$

It is directly evaluated in 4×4 block form:

$$J_{\nu}^{b}{}_{a'}^{\mu'} = \begin{bmatrix} \delta_{\nu}^{\mu} - \frac{i}{f} \partial_{\nu}\bar{\lambda}\gamma^{\mu}\theta & -\dfrac{\partial_{\nu}\bar{\lambda}_a}{f} \\ \\ \frac{i}{f} (\gamma^{\mu}\lambda)^{b} & \delta_{a}^{b} \end{bmatrix} \tag{2.20}$$

Its determinant is also directly reduced to the determinant of a 4×4 matrix, through standard block reduction:

$$\begin{aligned} \det J &= \det\left[\delta_{\nu}^{\mu} - \frac{i}{f} \partial_{\nu}\bar{\lambda}\gamma^{\mu}\theta + \frac{i}{f^2} \partial_{\nu}\bar{\lambda}\gamma^{\mu}\lambda\right] \\ &= \det W_{\kappa}^{\nu}(\delta_{\mu}^{\kappa} - \frac{i}{f} \nabla^{\kappa}\bar{\lambda}\gamma_{\mu}\theta) \\ &= |W| \det(\delta_{\mu}^{\kappa} - \frac{i}{f} \nabla^{\kappa}\bar{\lambda}\gamma_{\mu}\theta) \ . \end{aligned} \tag{2.21}$$

Significantly, to zeroth order in θ, this is just the V.A. determinant; so that the V.A. Lagrangian (2.10) arises simply from the following superspace volume component

$$-\frac{f^2}{2}\int d^4x' d^4\theta' \ \frac{(\bar{\theta}\theta)^2}{4} \ . \tag{2.22}$$

Since all expansions terminate after the fourth order in any spinor, the supertransformation (2.18) may be inverted completely, iteratively in θ' and $\lambda(x')$. Suppressing f:[7]

$$x_\mu = x'_\mu + i\bar{\lambda}(x')\gamma_\mu\theta' + \bar{\theta}'\gamma_\mu\partial'_\rho\lambda(x')\bar{\lambda}(x')\gamma_\rho\theta' + \ldots$$

$$\theta = \lambda(x') + \theta' + \frac{i}{f}\,\bar{\lambda}(x')\ \gamma_\mu\theta' + \ldots \tag{2.23}$$

The Jacobian of this transformation may be obtained indirectly, but easily, by inverting (2.20), as a function of the unprimed coordinates. This is the more useful form that we will rely on in the next section.

On the basis of supertranslation (2.18), Ivanov and Kapustnikov[7] noted that any linear superfield $\Phi(x,\theta)$ may be split into a completely decoupled array $\tilde{\Phi}$ of spectator fields:

$$\tilde{\Phi}(x,\theta,\lambda(x)) \equiv \Phi(x',\theta') = \Phi(x-i\bar{\lambda}(x)\gamma\theta,\theta-\lambda(x)) \ . \tag{2.24}$$

Indeed, the entire array transforms as a spectator. This follows by use of (2.1) and the transformations of the components of Φ, summarized by a supertranslation of x',θ':

$$\delta\tilde{\Phi}(x,\theta,\lambda(x)) = \Phi(x'_\mu + i\bar{\varepsilon}\gamma_\mu\theta' - i\delta\bar{\lambda}\gamma_\mu\theta, \theta' + \varepsilon - \delta\lambda) - \Phi(x'_\mu, \theta')$$

$$= \left[i(\bar{\varepsilon}-\delta\bar{\lambda})\gamma_\mu\theta - i\bar{\varepsilon}\gamma_\mu\lambda\right]\partial'_\mu\Phi(x',\theta') + (\varepsilon-\delta\lambda)\frac{\partial}{\partial\theta'}\Phi(x',\theta')$$

$$= -i\,\bar{\varepsilon}\gamma_\mu\lambda\left(\partial'^\mu - i\partial^\mu\bar{\lambda}\gamma_\kappa\theta\partial'^\kappa - \partial^\mu\lambda\frac{\partial}{\partial\theta'}\right)\Phi(x',\theta')$$

$$= \xi_\mu\partial^\mu\tilde{\Phi}(x,\theta,\lambda(x)) \quad . \tag{2.25}$$

The components of $\tilde{\Phi}$ do not transform into each other: they comprise the fully reduced representation of the Poincaré subgroup still acting linearly on all fields.[8] Consequently, the supersymmetry of the theory involved will not be affected if we constrain any components of $\tilde{\Phi}$ to be equal to a constant. In particular, λ--which so far has been set apart from the other fields and appears as an extraneous parameter--can take the place of the spinor field involved in the breaking of a spinor charge. This possibility proves useful in the derivation of effective Lagrangians described presently.

III. TRANSFORMING THE LINEAR LAGRANGIANS INTO THEIR NONLINEAR EQUIVALENT.

Consider the following toy model in two dimensions,[13] involving a real scalar A and a Majorana spinor ψ:

$$\mathcal{L}_{lin} = \frac{1}{2}\partial_\mu A\partial^\mu A + \frac{i}{2}\bar{\psi}\not{\partial}\psi + fA\bar{\psi}\psi - \frac{f^2}{2}(A^2+1)^2 \quad . \tag{3.1}$$

This is invariant under the supersymmetry transformations:

$$\delta A=\bar{\varepsilon}\psi \ , \ \delta\psi=[f(1+A^2) - i\not{\partial}A]\varepsilon \tag{3.2}$$

where f is a constant of dimension 1. The v.e.v. of the scalar is $\langle A\rangle=0$ and hence the spinor is massless, in contrast to the scalar, which has mass $m=\sqrt{2}f$. Supersymmetry is thus broken at the tree level: $\langle\delta\psi\rangle=f\varepsilon$, and the vacuum energy density is $f^2/2$. The supercurrent is:

$$S_\mu = [if(1+A^2) - \partial_\nu A\gamma^\nu]\gamma_\mu\psi \tag{3.3}$$

Note that, in contrast to regular symmetries, a potential of the form $(A^2-1)^2$ would <u>not</u> break supersymmetry. In our particular model it turns out that supersymmetry breaking is not invalidated by radiative corrections [14]. Even if it were, this would still not be at issue here. We wish to stress again that our semiclassical reasoning is illustrative of the situation prevailing in an arbitrary number of dimensions, including four, whether supersymmetry is broken or not. (Still, the nonlinear Lagranians will be useful only when supersymmetry is broken. We discuss this later).

We proceed to transform this model to an equivalent effective Lagrangian of the general type outlined in the previous section. However, since the algebraic aspects of supersymmetry are significant at this stage, we must restore (3.1)-(3.2) to a form in which the transformations (3.2) close into the sypersymmetry algebra without use of the spinor equations of motion. We thus regress to the original superfield formulation of this theory:

$$S=\int d^2x\,\mathcal{L}_{lin} = \frac{1}{2}\int d^2x\ d\bar{\theta}d\theta\left[\frac{1}{2}\ \overline{D\Phi}D\Phi - 2f\left(\frac{\Phi^3}{3}+\Phi\right)\right] \tag{3.4}$$

$$=\int d^2x\left[\frac{1}{2}(\partial_\mu A)^2 + \frac{i}{2}\bar{\psi}\not{\partial}\psi + \frac{F\ F}{2} - Ff(1+A^2) + \bar{\psi}\psi fA\right]$$

where $\Phi(x,\theta) = A + \bar{\theta}\psi + \frac{1}{2}\ \bar{\theta}\theta F$ (3.5)

$$\delta\Phi(x,\theta) = \Phi(x_\mu + i\bar{\varepsilon}\gamma_\mu\theta, \theta+\varepsilon) - \Phi(x,\theta) \tag{3.6}$$

and $D \equiv \frac{\partial}{\partial\bar{\theta}} - i\not{\partial}\theta$ (3.7)

In component form the supertranslation (3.6) reads:

$$\delta A=\bar{\varepsilon}\psi \qquad \delta\psi=(F-i\not{\partial}A)\varepsilon \qquad \delta F=-i\bar{\varepsilon}\not{\partial}\psi \tag{3.8}$$

These transformations close into the supersymmetry algebra. The "physical" formulation (3.1), (3.2) follows by simply eliminating the scalar auxiliary field F through its algebraic equations of motion:

$$F=f(1+A^2) \tag{3.9}$$

Observe that $\langle F\rangle=f$.

Suppressing f, we first evaluate the components of $\tilde{\Phi}$ as defined through (2.24) and (3.5):

$$\tilde{A} = A - \bar{\lambda}\psi + \frac{F}{2}\bar{\lambda}\lambda$$

$$\tilde{\psi} = \psi - \lambda F + i\partial\!\!\!/ A\lambda + \frac{i}{2}\,\partial\!\!\!/\psi\bar{\lambda}\lambda \tag{3.10}$$

$$\tilde{F} = F + i\bar{\lambda}\partial\!\!\!/\psi - \partial^2 A\,\frac{\bar{\lambda}\lambda}{2}$$

It is straightforward to check that $(\tilde{A},\tilde{\psi},\tilde{F})$ indeed transform as spectators. The transformation (3.10) may be easily inverted to yield the following expressions (involving derivatives of λ, in contrast to 3.10):

$$A = \tilde{A} + \bar{\lambda}\tilde{\psi} + \frac{\tilde{F}}{2}\,\bar{\lambda}\lambda$$

$$\psi = \tilde{\psi} + \tilde{F}\lambda - i\partial\!\!\!/\tilde{A}\lambda - i\lambda\tilde{F}\bar{\lambda}\partial\!\!\!/\lambda - i\gamma_\mu\lambda\partial^\mu(\bar{\lambda}\tilde{\psi})$$

$$F = |W|\tilde{F} - i\bar{\lambda}\partial\!\!\!/\tilde{\psi} - \frac{1}{2}\,\bar{\lambda}\lambda\partial^2\lambda\tilde{\psi} - \frac{\bar{\lambda}\lambda}{2}\,\partial^2\tilde{A} - \bar{\lambda}\gamma_\nu\gamma_\mu\partial^\nu\lambda\partial^\mu\tilde{A}$$

$$- \bar{\lambda}\lambda\partial^\mu\bar{\lambda}\partial_\mu\tilde{\psi} - \bar{\lambda}\gamma_\mu\gamma_\nu\partial^\mu\lambda\partial^\nu(\bar{\lambda}\tilde{\psi}) \tag{3.11}$$

where (cf. 2.6):

$$|W| = 1 - i\bar{\lambda}\partial\!\!\!/\lambda - \frac{\bar{\lambda}\lambda}{2}\,\varepsilon^{\mu\nu}\partial_\mu\bar{\lambda}\gamma_5\partial_\nu\lambda \tag{3.12}$$

Now since the supersymmetry transformations of $(\tilde{A},\tilde{\psi},\tilde{F})$ are not coupled together, any or all of them may be set equal to a constant consistent with the vacuum values of the fields on the right hand side of (3.10). If, for instance

$\tilde{A}=\tilde{\psi}=0$, $\tilde{F}=1$, the original fields (A,ψ,F) are nontrivially constrained (as in the nonlinear σ-model) and eqs.(3.11),(3.4) reproduce the expressions of Ref. [11], the analogue of nonlinear pion effective Lagrangians.[9]* If, on the other hand, we do not wish to constrain away any original degrees of freedom, we could still set $\tilde{\psi}=0$ (only) without any loss of dynamical information. This is because the "angular" degree of freedom is already carried by the Goldstone field λ, and $\tilde{\psi}$ corresponds to the arbitrary origin in "rotation" space and not to an independent degree of freedom. $\tilde{\psi}=0$ in (3.10) provides an equation for λ in terms of (A,ψ,F):

$$\lambda = G\left[\psi + \frac{i}{2}\,\not{\partial}\psi\;\bar{\psi}\bar{G}G\psi\right] \tag{3.13}$$

where G is defined implicity in terms of (A,ψ,F):

$$(F - i\,\not{\partial}A)G = \mathbb{1}\;. \tag{3.14}$$

As a result, (3.10) and (3.11) reduce to the invertible transformation connecting (A,ψ,F) with $(\tilde{A},\lambda,\tilde{F})$:

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*We note in passing that the constraints of Ref. [11] as expressed in Eq. (9a) obscure the essential irrelevance of $\Phi^2=c$ to the breaking of supersymmetry. It may, or course, be imposed in the supersymmetric σ-model without supersymmetry breaking. It is also, for c=0, strictly a consequence of the following alternative we might propose: $1/2\,\bar{D}\Phi D\Phi=\Phi$, which has components $A=1/2\,\bar{\psi}\psi$, $F=i\bar{\psi}\not{\partial}\psi+F^2+\partial A\cdot\partial A$, and $(F+i\not{\partial}A)\psi=\psi$. The second component may be solved for F; the solution $F=1-i\bar{\psi}\not{\partial}\psi-1/2\bar{\psi}\psi\partial_\mu\bar{\psi}\partial^\mu\psi+\bar{\psi}\not{\partial}\psi\bar{\psi}\not{\partial}\psi$ which breaks supersymmetry ($\langle F\rangle=1$) is chosen by the third component, which otherwise doesn't seem to eliminate any further degrees of freedom. These constraints appear remarkably simpler $(\tilde{A}=0,\tilde{F}=1)$ in the reduced realization.

$$\tilde{A} = A - \bar{\psi}(\bar{G} - \frac{F}{2}\,\bar{G}G)\psi$$

$$\lambda = G\psi + \frac{i}{2}\,G\not{\partial}\psi\bar{\psi}\bar{G}G\psi$$

$$\tilde{F} = F - i\partial_\mu\bar{\psi}\gamma^\mu(G\psi + \frac{i}{2}\,G\not{\partial}\psi\bar{\psi}\bar{G}G\psi) - \frac{\partial^2 A}{2}\,\bar{\psi}\bar{G}G\psi \tag{3.15}$$

$$A = \tilde{A} + \frac{\tilde{F}}{2}\,\bar{\lambda}\lambda$$

$$\psi = \frac{i}{2}\,\tilde{F}\,\not{\partial}\lambda\,\bar{\lambda}\lambda - i\not{\partial}\tilde{A}\lambda + \tilde{F}\lambda$$

$$F = \tilde{F}|W| - \partial^2\tilde{A}\,\frac{\bar{\lambda}\lambda}{2} - \bar{\lambda}\gamma^\mu\gamma^k\partial_k\tilde{A}\partial_\mu\lambda\,. \tag{3.16}$$

One might, at the expense of tedium, substitute (3.16) into (3.4), to obtain the nonlinear Lagrangian. We note however that it is far easier to work with superfields. To illustrate the essential redundance of $\tilde{\psi}$ discussed above, we retain it in the rest of this section and only drop it in the end; we will exemplify in the next section how its presence doesn't change the theory.

We first observe that, from (2.24), $\Phi(x',\theta')=\tilde{\Phi}(x,\theta,\lambda(x))$, $d^2x'd^2\theta' = d^2xd^2\theta|J(x,\theta,\lambda(x))|$, and D' is expressible in terms of the unprimed coordinates, given $J^{-1}(x,\theta,\lambda(x))$. As a result, (3.4) with its integration variables primed is directly recast into the nonlinear Lagrangian with unprimed variables. Specifically, the inverse of (2.20) is just:

$$J^{-1a'\kappa}_{\mu'c} = \begin{bmatrix} V_\mu{}^\kappa & \partial_\rho\bar{\lambda}_c V_\mu{}^\rho \\ \\ -i(\gamma_\rho\lambda)^a\ V_\rho{}^\kappa & \delta^a_c - i(\gamma^\nu\lambda)^a\partial_\rho\bar{\lambda}_c V_\nu{}^\rho \end{bmatrix} \tag{3.17}$$

where: $$(\delta^\mu_\nu + T^\mu_\nu - i\partial_\nu\bar{\lambda}\gamma^\mu\theta)V_\mu{}^\kappa = \delta^\kappa_\nu \tag{3.18}$$

so that the "supervierbein" $V_\mu{}^\kappa$ is easily solved to be

$$V_\mu{}^\kappa = (W^{-1})^\kappa_\mu + i\ (W^{-1})^\kappa_\rho\ \nabla_\mu\bar{\lambda}\gamma^\rho\theta + O(\theta^2)\,. \tag{3.19}$$

In consequence,

$$D' = \frac{\partial}{\partial\bar{\theta}'} - i\gamma^\mu(\theta-\lambda)\partial'_\mu = (1 - i\gamma^\mu\theta\partial_\nu\bar{\lambda}V_\mu^\nu)\ \frac{\partial}{\partial\bar{\theta}} - i\gamma^\mu\theta V_\mu^\kappa\partial_\kappa\,. \tag{3.20}$$

We further observe that

$$V_\mu^\nu\partial_\nu = \nabla_\mu - i\bar{\theta}\gamma_\kappa\nabla_\mu\lambda\nabla^\kappa + O(\theta^2) \tag{3.21}$$

and $$|J| = |W|[1 + i\bar{\theta}\gamma\cdot\nabla\lambda - \frac{\bar{\theta}\theta}{2}\,\varepsilon^{\mu\nu}\nabla_\mu\bar{\lambda}\gamma_5\nabla_\nu\lambda]\,. \tag{3.22}$$

The superfield integrands are

$$\frac{1}{2}\,\overline{D\tilde{\Phi}}D\tilde{\Phi} = \frac{\bar{\tilde{\psi}}\tilde{\psi}}{2} - i\bar{\tilde{\psi}}\gamma^{\mu}\theta V_{\mu}^{\nu}(\partial_{\nu}\tilde{A} + \partial_{\nu}\bar{\lambda}\tilde{\psi}) + \bar{\tilde{\psi}}\theta\tilde{F}$$

$$+ \frac{\bar{\theta}\theta}{2}\left[i\bar{\tilde{\psi}}\gamma^{\mu}(\nabla_{\mu}\tilde{\psi} + \nabla_{\mu}\lambda\tilde{F}) + \tilde{F}^{2} + (\nabla_{\mu}\tilde{A} + \nabla_{\mu}\bar{\lambda}\tilde{\psi})^{2}\right] \qquad (3.23)$$

$$-2\left(\frac{\tilde{\Phi}^{3}}{3} + \tilde{\Phi}\right) = -2\left[\tilde{A} + \frac{\tilde{A}^{3}}{3} + \bar{\theta}\tilde{\psi}(1+\tilde{A}^{2}) + \frac{\bar{\theta}\theta}{2}\left(\tilde{F}(1+\tilde{A}^{2}) - \bar{\tilde{\psi}}\tilde{\psi}\tilde{A}\right)\right]. \qquad (3.24)$$

Inserting the above into (3.4) with primed dummy integration variables, changing variables from primed to unprimed, integrating out $\theta, \bar{\theta}$, and eliminating $\tilde{F}$ through its equations of motion, which are again $\tilde{F}=(1+\tilde{A}^{2})$, we are led to the following V.A. Lagrangian:

$$S = \int d^{2}x\,|W|\left[\frac{1}{2}\nabla_{\mu}\tilde{A}\nabla^{\mu}\tilde{A} - \frac{1}{2}(1+\tilde{A}^{2})^{2} + \left(\tilde{A} + \frac{\tilde{A}^{3}}{3}\right)\varepsilon^{\mu\nu}\nabla_{\mu}\bar{\lambda}\gamma_{5}\nabla_{\nu}\lambda\right.$$

$$+ \frac{i}{2}\bar{\tilde{\psi}}\gamma\cdot\nabla\tilde{\psi} + \tilde{A}\bar{\tilde{\psi}}\psi + \frac{1}{2}(\nabla_{\mu}\bar{\lambda}\tilde{\psi})^{2} + \nabla_{\mu}\tilde{A}\nabla^{\mu}\bar{\lambda}\tilde{\psi}$$

$$\left. + i\bar{\tilde{\psi}}\gamma\cdot\nabla\lambda(1+\tilde{A}^{2}) + \varepsilon^{\mu\nu}\bar{\tilde{\psi}}\gamma_{5}\nabla_{\mu}\lambda\nabla_{\nu}\tilde{A} + \frac{\bar{\tilde{\psi}}\tilde{\psi}}{4}\varepsilon^{\mu\nu}\nabla_{\mu}\bar{\lambda}\gamma_{5}\nabla_{\nu}\lambda\right].$$

$$(3.25)$$

We may now set $\tilde{\psi}=0$ and retain only the first line of this nonlinear Lagrangian, which has the general form restricted in the previous section, and is thus manifestly supersymetric. Naturally, if we set $\lambda=0$ instead, we reproduce (3.1).

IV. TREE AMPLITUDES AND LOW ENERGY BEHAVIOR

The nonlinear Lagrangian derived in the previous section:

$$\mathcal{L}_{\text{nonlin}} = |W|\left[\frac{1}{2}\,\nabla_\mu\tilde{A}\nabla^\mu\tilde{A} - \frac{1}{2}\,f^2(1+\tilde{A}^2)^2 + \frac{1}{f}\,(\tilde{A} + \frac{\tilde{A}^3}{3})\,\varepsilon^{\mu\nu}\nabla_\mu\bar{\lambda}\gamma_5\nabla_\nu\lambda\right]$$

$$= \frac{i}{2}\,\bar{\lambda}\not\partial\lambda + \frac{1}{2}\,\partial_\mu\tilde{A}\partial^\mu\tilde{A} - f^2\tilde{A}^2 - \frac{f^2}{2} + \frac{\tilde{A}}{f}\,\varepsilon^{\mu\nu}\partial_\mu\bar{\lambda}\gamma_5\partial_\nu\lambda +$$

$$+ i\tilde{A}^2\bar{\lambda}\not\partial\lambda + \frac{\bar{\lambda}\lambda}{4f^2}\,\varepsilon^{\mu\nu}\partial_\mu\bar{\lambda}\gamma_5\partial_\nu\lambda + \frac{i}{f^2}\,\partial_\mu\tilde{A}\partial_\nu\tilde{A}\bar{\lambda}\gamma^\nu\partial^\mu\lambda + \ldots \tag{4.1}$$

is equivalent to the linear one (3.1) via the nonlinear redefinitions (3.15), (3.16)- and could thus be transformed back to it. After the F's are eliminated, the transformations start with a linear term ($\lambda=\psi+\ldots,\tilde{A}=A+\ldots$). As a result, the loop expansion parametric weight of the fields contributing to a tree amplitude is the same for both linear and nonlinear models.[8] Hence the S-matrix amplitudes (i.e. on-shell) for the two theories coincide at the tree level--"the phenomenological approximation".[9] This is easily seen in a few simple amplitudes; (ψ,A) and $(\lambda,\tilde{A})$ serve as equally valid interpolating fields for the massless fermion and the massive scalar ($m=\sqrt{2}f$).

The tree approximation to the decay of a scalar to the fermions (in its rest frame the fermion momenta are $\pm m/2=\pm f/\sqrt{2}$) is the same for the linear and the nonlinear models respectively:

$$<f_1(p)f_2(-p)|\int d^2x f\bar{\psi}(x)\psi(x)A(x)|S(0)> =$$

$$= <0|\ \frac{f}{2}\ (\bar{u}(-p)u(p)\ -\ \bar{u}(p)u(-p)|0> = if \tag{4.2}$$

$$<f_1(p)f_2(-p)|\int d^2x\ \frac{\varepsilon^{\mu\nu}}{f}\ \partial_\mu\bar{\lambda}\gamma_5\partial_\nu\lambda\tilde{A}|S(0)> =$$

$$= <0|\ \frac{2p^2}{2f}\ (\bar{u}(-p)u(p)\ -\ \bar{u}(p)u(-p))|0> = if \tag{4.3}$$

Conventions and normalizations may be found in the Appendix. In fact, the same amplitude is also obtainable from (3.25), i.e. even before the elimination of $\tilde{\psi}$. In that case, $\tilde{\psi}-\lambda$ would have no kinetic term, while $\tilde{\psi}+\lambda$ would, and therefore the latter combination would be the interpolating field for the massless fermion. This leads to an amplitude:

$$<f_1(p)f_2(-p)|\int d^2x \Bigg[f\tilde{A}\bar{\tilde{\psi}}\tilde{\psi} + \frac{\varepsilon^{\mu\nu}}{f}\ \partial_\mu\bar{\lambda}\gamma_5\partial_\nu\lambda\tilde{A}\ +$$

$$+\left(\partial^\mu\bar{\lambda}+\varepsilon^{\mu\nu}\partial_\nu\bar{\lambda}\gamma_5\right)\frac{\tilde{\psi}}{f}\ \partial_\mu\tilde{A}\Bigg]|S(0)> = if \tag{4.4}$$

precisely as before.*

*If the term 2Φ in the potential of (3.4) were absent, supersymmetry would break neither in the linear, nor in the nonlinear model, as evidenced in the vacuum energies. Moreover (3.15)-(3.16) would not, in this case, relate the fields through a linear term plus multilinear corrections, and the free amplitudes of the two models would not coincide. For instance, the nonlinear model would clearly not have a scalar-bispinor coupling, in contrast to the linear one, and thus the effective Lagrangian would not be "phenomenological" in the above sense.

A more intricate "conspiracy" of diagrams is observed in the identical tree amplitudes for fermion-fermion scattering. For example, in the center of momentum frame (all fermions carry energy p), the forward amplitude (in two dimensions the only alternative to this is backward scattering) is given by boson exchange in the linear model (Fig. 1); and by both boson exchange (Fig. 1) and four-Fermi contact interactions (Fig. 2) in the nonlinear one. The linear model leads to the tree amplitude:

$$f^2\left(\frac{1}{m^2-4p^2} - \frac{1}{m^2+4p^2}\right) = \frac{4p^2/m^2}{1-16p^4/m^4} \tag{4.5}$$

The boson exchange diagrams for the nonlinear model give

$$\frac{1}{f^2} \cdot 4p^4\left(\frac{1}{m^2-4p^2} - \frac{1}{m^2+4p^2}\right) = \frac{64p^6/m^6}{1-16p^4/m^6} \tag{4.6}$$

The contact coupling contribution (Fig. 2) is

$$4 \cdot \frac{1}{4f^2} \cdot 2p^2 = 4p^2/m^2 \tag{4.7}$$

Thus the overall tree amplitude (4.6) + (4.7) for the nonlinear theory is

$$\frac{4p^2/m^2}{1-16p^4/m^4} \tag{4.8}$$

identical to (4.5). Technical details are provided in the Appendix. The amplitudes vanish for $p \to 0$. Similar results hold for backward scattering.

As a last example, we compute the fermion-scalar scattering amplitude. In the linear theory only the tree diagrams of Fig. 3 a,b contribute, while in the nonlinear model there is an additional contribution from a contact term (Fig. 3c). Consider forward scattering in the boson rest frame (fermion momentum/energy ω). The tree amplitude for the linear model is:

$$2f^2 m\left(\frac{1}{m(m+2\omega)} - \frac{1}{m(m-2\omega)}\right) = \frac{-4\omega}{1-4\omega^2/m^2} \qquad (4.9)$$

The amplitude for the nonlinear model is:

$$-2\,\frac{m^3\omega^2}{f^2}\left(\frac{1}{m(m+2\omega)} - \frac{1}{m(m-2\omega)}\right) - 2\,\frac{m^2\omega}{f^2} = \frac{-4\omega}{1-4\omega^2/m^2} \qquad (4.10)$$

The amplitudes coincide and vanish as $\omega \to 0$ (Adler decoupling; observe that in Lagrangian (4.1) the interactions of λ are all given by derivative couplings). The amplitude for backward scattering is identically zero.

The low energy behavior of amplitudes like the above may alternatively be determined without reference to a particular Lagrangian, through current algebra techniques. As an example, we turn to the soft spinor theorems of the Adler-Weisberger type[2,3] relevant to the above process (in analogy to π-N scattering of chiral dynamics). The matrix element for two supercurrents interacting with a scalar state is (Fig. 4)

$$M^{ab}_{\mu\nu} = i\int d^2x e^{iq\cdot x} \langle A_f | T\left(S^a_\mu(x)\ \bar{S}^b_\nu(0)\right) | A_i\rangle \tag{4.11}$$

The following Ward identity follows immediately:

$$\begin{aligned} q^\mu M_{\mu\nu} &= -\int d^2x e^{iq\cdot x}\Big(\langle A_f | T(\partial\cdot S^a(x)\bar{S}^b_\nu(0))\ |A_i\rangle + \\ &\qquad + \delta(x^0)\langle A_f | \{S^a_o(x), \bar{S}^b_\nu(0)\} | A_i\rangle\Big) = \\ &= -2\gamma^\mu \langle A_f | \partial_\mu A \partial_\nu A - g_{\mu\nu}\left(\frac{\partial_\kappa A \partial^\kappa A}{2} - \frac{f^2}{2}\ (1+A^2)^2\right) | A_i\rangle = \\ &= -2\gamma^\mu \langle A_f | \Theta_{\mu\nu}(0)\ | A_i\rangle \end{aligned} \tag{4.12}$$

where we have used the conservation law for supercurrents and their equal-time anticommutation relations, e.g. as defined in (3.3):

$$\begin{aligned} \delta(x^0)\{S_o(x), \bar{S}_\nu(0)\} &= 2\delta^2(x)\gamma^\mu\left[\partial_\nu A \partial_\mu A - g_{\mu\nu}\left(\frac{\partial_\kappa A \partial^\kappa A}{2} - \frac{f^2}{2}(1+A^2)^2\right)\right] \\ &\quad + \delta^2(x) 2f\varepsilon_{\mu\nu}\gamma_5 \partial^\mu\left(A + \frac{A^3}{3}\right) + O(\psi^2) \end{aligned} \tag{4.13}$$

plus, conceivably, irrelevant Schwinger terms. The term in the bracket is the bosonic contribution to the energy-momentum tensor $\Theta_{\mu\nu}$, in accordance with the charge algebra, which is of course the integrated version of (4.13). The next term (curl) is a trivially conserved, chargeless "improvement," characteristic of the current supermultiplet under consideration,[15] and vanishes in the matrix element (4.12).

The amputated amplitude we are interested in is the matrix element (between on shell fermion states) of the residue $M^{(0)}$ at the Goldstone poles[2,3] in the Green's function $M_{\mu\nu}$:

$$M^{ab}_{\mu\nu} = f^2\left[\gamma_\mu \frac{1}{\not q} M^{(0)} \frac{1}{\not \ell} \gamma_\nu\right]^{ab} + \text{(less singular terms as } q,\ell\to 0) \tag{4.14}$$

Hence the low energy limit of $M^{(0)}$ may be read off from the low energy limit of $M_{\mu\nu}$:

$$\frac{1}{f^2}\, q^\mu M_{\mu\nu} \ell^\nu = M^{(0)}, \text{ as } q,\ell\to 0 \ . \tag{4.15}$$

The matrix element of $M^{(0)}$ for forward scattering $(q=l=(\omega,\omega),\ p_i=p_f=(m,0))$ is obtained from (4.12):

$$\bar{u}(q)\gamma_\mu u(\ell)\,\frac{(q+\ell)^\nu}{f^2}\ \langle A_f|-p_\mu p_\nu + g_{\mu\nu}\left(\frac{p_\kappa p^\kappa}{2} + \frac{f^2}{2}\ (1+A^2)\right)|A_i\rangle =$$

$$= \frac{-4mp\cdot q}{m^2} = -4\omega \ , \tag{4.16}$$

which is checked to be the low energy limit of (4.9)-(4.10), and displays decoupling as $\omega\to 0$. We refer the reader to Refs. [2,3] for more details.

A given current algebra determines a particular effective Lagrangian which incorporates the low energy theorems derivable from it. The systematic approach involved in this problem outranges our present scope: we do not attempt to provide a strict proof for the universality of the Volkov-Akulov Lagrangian.

V. GENERALIZATION AND DISCUSSION

The general principle for connecting linear and phenomenological Lagrangians is evident from Section III: First, the linear theory is expressed in superfield form. The argument of the superfields Φ is then shifted by $-\lambda(x)/f$ in superspace, thereby producing the fully reduced arrays $\tilde{\Phi}$. A number of spectator fermions involved in supersymmetry breaking may be eliminated in favor of an equal number of ancillary Volkov-Akulov fermions λ. This results in no loss of information and significantly simplifies the subsequent conversion of the theory to its nonlinear correspondent. If desirable, the nonlinear theory may be linearized back to the original form, provided no actual degrees of freedom have been relinquished.

The effective Lagrangians described are useful mostly in the context of broken supersymmetry, since only then can the V.A. fields λ be interpreted as the massless fermions of the theory: without a constant term multiplying the V.A. determinant, there can be no net kinetic term for λ in the phenomenological Lagrangian. Useful or not however, they constitute mere formal relabelings of the original linear theories and do not display new symmetry properties. If supersymmetry is broken, the effective Lagrangian language sets the Goldstone ("angular") degrees of freedom apart from the others and makes the structure of some low energy effects more transparent.

We could further illustrate our procedure through more extensive examples (gauge multiplets,[13] higher dimensional theories, etc.) were it not for the proliferation of multiplets required for spontaneous supersymmetry breaking at the tree level.[1] The effective Larangians discussed should also, in principle, lend themselves to the study of dynamical supersymmetry breaking as well,[4] but we will not discuss the problem here, in the absence of simple paradigms, e.g. comparable to that of Section III.

For the rest of the paper we confine ourselves to detailing the extension of our algorithm to extended supersymmetry and constrained superfields, which are most common in usual applications. In full analogy to (2.1), the following realization may be written[12] for the algebra of N spinorial charges:

$$\delta\lambda^j = f\varepsilon^j - \frac{i}{f}\,\partial_\mu\lambda^j\,\bar{\varepsilon}^k\gamma^\mu\lambda^k \ , \tag{5.1}$$

$j,k=1,\dots,N$. Summation over repeated indices is implied. The central charge is not represented even though there is a dimensional parameter f available. Observe that each spinor λ^j transforms nonlinearly with respect to the parameter ε^j corresponding to the associated generator broken, but, in addition, it transforms as a bonafide spectator with respect to all remaining parameters.

The discussion of Sect. II readily carries through upon redefinition of the O(N) singlet matrix:

$$T_\mu{}^\nu \equiv -\frac{i}{f^2}\bar{\lambda}^j\gamma_\mu\partial^\nu\lambda^j \quad . \tag{5.2}$$

Note that, because now there are more spinors involved, several expansions, like that of $|J|$ or $W_\mu^{-1\nu}$, but not of $|W|$, will terminate at generally higher orders in the λ's and θ's. $|W|$ is obtainable from the Jacobian of the N-superspace translation:

$$x'_\mu = x_\mu - \frac{i}{f}\bar{\lambda}^j(x)\gamma_\mu\theta^j$$

$$\theta'^j = \theta^j - \frac{\lambda^j(x)}{f} \; . \tag{5.3}$$

The pure V.A. Lagrangian is at least O(N) symmetric. Spectators are accommodated in the usual manner.

We proceed to illustrate the reduction procedure through the two-dimensional, N=2 scalar multiplet[13] (which is also obtainable through dimensional reduction[16] of the N=1, 4-dimensional scalar multiplet). We rename: $\theta^1,\varepsilon^1,\lambda^1\to\theta,\varepsilon,\lambda$ and $\theta^2,\varepsilon^2,\lambda^2\to\hat{\theta},\hat{\varepsilon},\hat{\lambda}$, to distinguish their group indices from those of the matter fields A^m,ψ^m,F^m, where m=1,2. The matter fields rotate into each other through the O(2) transformation $A^m\to\hat{A}^m\equiv\varepsilon^{mn}A^n$ (note $\hat{\hat{}}=-1$). Suppressing these O(2) indices as well as f in what follows, we may write the relevant N=2 superfield:[13]

$$\Phi = A + \bar{\theta}\gamma + \bar{\underset{\wedge}{\theta}}\hat{\psi} + \frac{1}{2}(\bar{\theta}\theta - \bar{\underset{\wedge}{\theta}}\underset{\wedge}{\theta})F + \bar{\underset{\wedge}{\theta}}\theta\hat{F} + i\bar{\theta}\not{\partial}\hat{A}\underset{\wedge}{\theta}$$

$$+ \frac{i}{2}(\bar{\theta}\theta + \bar{\underset{\wedge}{\theta}}\underset{\wedge}{\theta})(\bar{\theta}\not{\partial}\psi + \bar{\underset{\wedge}{\theta}}\not{\partial}\hat{\psi}) + \frac{1}{4}\bar{\theta}\theta\bar{\underset{\wedge}{\theta}}\underset{\wedge}{\theta}\partial^2 A . \qquad (5.4)$$

The components of the superfield variation $\delta\Phi$ entailed by a superspace transformation $\delta x^\mu = i(\bar{\varepsilon}\gamma_\mu\theta + \bar{\underset{\wedge}{\varepsilon}}\gamma_\mu\underset{\wedge}{\theta})$, $\delta\theta = \varepsilon$, $\delta\underset{\wedge}{\theta} = \underset{\wedge}{\varepsilon}$, are:

$$\delta A = \bar{\varepsilon}\psi + \bar{\underset{\wedge}{\varepsilon}}\hat{\psi}$$

$$\delta\psi = -i\not{\partial}A\varepsilon + i\not{\partial}\hat{A}\underset{\wedge}{\varepsilon} + F\varepsilon + \hat{F}\underset{\wedge}{\varepsilon}$$

$$\delta F = -i\bar{\varepsilon}\not{\partial}\psi + i\bar{\underset{\wedge}{\varepsilon}}\not{\partial}\hat{\psi} . \qquad (5.5)$$

Note that two copies of the N=1 model of Section III are recoverable upon suppression of ε and θ, or $\underset{\wedge}{\varepsilon}$ and $\underset{\wedge}{\theta}$. The higher components of the superfield (5.4) are related to the lower ones through the supersymmetric constraint:

$$D\Phi + \underset{\wedge}{D}\hat{\Phi} = 0 . \qquad (5.6)$$

As outlined in Section II, the standard reduced array $\tilde{\Phi}$ is readily obtained by a $\lambda(x)$-dependent supertranslation of Φ:

$$\tilde{\Phi} = \Phi(x - i\bar{\lambda}\gamma\theta - i\bar{\underset{\wedge}{\lambda}}\gamma\underset{\wedge}{\theta}, \theta - \lambda(x), \underset{\wedge}{\theta} - \underset{\wedge}{\lambda}(x)) . \qquad (5.7)$$

The spectator transformation law (2.25) follows *mutatis mutandis*, with $\xi_\mu = -i(\bar{\varepsilon}\gamma_\mu\lambda + \bar{\underset{\wedge}{\varepsilon}}\gamma_\mu\underset{\wedge}{\lambda})$.

As in the constrained N=1 superfield $[\bar{\psi}\psi/2+\bar{\theta}\psi+\bar{\theta}\theta(1-i\bar{\psi}\not\partial\psi+\ldots)/2]$ of Ref. [11], the constraint (5.6) may be enforced before or after the above reduction. The latter case may be technically preferable, since then the constraint pertains only to the higher components of the reduced array $\tilde{\Phi}$ (cf. footnote p. 17). The essential components are thus only the lower ones:

$$\tilde{A} = A - \bar{\lambda}\psi - \hat{\bar{\lambda}}\hat{\psi} + i\bar{\lambda}\not\partial\hat{A}\hat{\lambda} + \frac{1}{2}\,(\bar{\lambda}\lambda-\hat{\bar{\lambda}}\hat{\lambda})F + \hat{\bar{\lambda}}\lambda\hat{F}$$

$$- \frac{i}{2}\,(\bar{\lambda}\lambda+\hat{\bar{\lambda}}\hat{\lambda})(\bar{\lambda}\not\partial\psi+\hat{\bar{\lambda}}\not\partial\hat{\psi}) + \frac{1}{4}\,\bar{\lambda}\lambda\;\hat{\bar{\lambda}}\hat{\lambda}\partial^2 A$$

$$\tilde{\psi} = \psi + i\not\partial A\lambda - i\not\partial\hat{A}\hat{\lambda} - \lambda F - \hat{\lambda}\hat{F} + \frac{1}{2}\,(\hat{\lambda}\bar{\lambda}\lambda\partial^2\hat{A} - \lambda\hat{\bar{\lambda}}\hat{\lambda}\partial^2 A)$$

$$+ \frac{i}{2}\,\not\partial\psi(\bar{\lambda}\lambda+\hat{\bar{\lambda}}\hat{\lambda}) + i(\lambda\hat{\bar{\lambda}}\not\partial\hat{\psi}-\gamma_\mu\lambda\hat{\bar{\lambda}}\partial^\mu\hat{\psi}) - \frac{\partial^2\psi}{4}\,\bar{\lambda}\lambda\hat{\bar{\lambda}}\hat{\lambda}$$

$$\tilde{F} = F - i\bar{\lambda}\partial\hat{F}\hat{\lambda} + \bar{\lambda}\hat{\lambda}\partial^2\hat{A} - \frac{1}{2}\,(\bar{\lambda}\lambda-\hat{\bar{\lambda}}\hat{\lambda})\partial^2 A + i\,(\bar{\lambda}\not\partial\psi-\hat{\bar{\lambda}}\not\partial\hat{\psi})$$

$$+ \frac{1}{2}\,(\bar{\lambda}\lambda+\hat{\bar{\lambda}}\hat{\lambda})(\hat{\bar{\lambda}}\partial^2\hat{\psi}-\bar{\lambda}\partial^2\psi)\;. \qquad (5.8)$$

Following the reasoning of Section III, $\tilde{\psi}$ may be set equal to zero, without any restriction of the dynamics. This, in turn, determines λ and $\hat{\lambda}$ in terms of (A,Ψ,F), for $F\cdot F\neq 0$:

$$\lambda = F \cdot \psi \, (F \cdot F)^{-1} + \ldots$$

$$\hat{\lambda} = \hat{F} \cdot \psi \, (F \cdot F)^{-1} + \ldots \qquad (5.9)$$

The dot product denotes contraction of the matter field group indices.

Expansion around a perturbative supersymmetry breaking vacuum value $\langle F \rangle \neq 0$ enables the nonlinear redefinitions (5.9) to start with a tern linear in the original fields, as was the case in Section III. In order to actualize this situation through a tree potential, the introduction of more multiplets might be necessary,[1] which is of course covered by the above discussion, but would involve longer manipulations.

Construction of such potentials is part of a technology which is outside the scope of this investigation. Nonetheless, the present work demonstrates that, given a satisfactory linear model, there are no conceptual difficulties involved in its conversion to the equivalent Volkov-Akulov form.

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APPENDIX: Conventions and Technical Definitions.

We employ Majorana spinors: $\psi = C\bar{\psi}^T$. Their bilinears have particularly convenient flip properties. In two dimensions:

$$\bar{\psi}\psi' = \bar{\psi}'\psi;\ \bar{\psi}\gamma_5\psi' = -\bar{\psi}'\gamma_5\psi;\ \bar{\psi}\gamma_\mu\psi' = -\bar{\psi}'\gamma_\mu\psi.$$

The Fierz transposition is simply:

$$\psi\bar{\psi}'\psi'' = -\frac{1}{2}\left(\psi''\bar{\psi}'\psi + \gamma_5\psi''\bar{\psi}'\gamma_5\psi + \gamma_\mu\psi''\bar{\psi}'\gamma^\mu\psi\right)$$

(e.g. check $\psi\bar{\psi}\psi = 0$, etc... for any trilinear of the same spinor).

In two dimensions, the gamma matrices satisfy: $\gamma^\mu\gamma^\nu = g^{\mu\nu}\mathbb{1} + \varepsilon^{\mu\nu}\gamma_5$ where our metric is timelike ($g^{00} = 1$), and $\varepsilon^{01} = 1$. In the Majorana representation, γ^0 and γ^1 are imaginary: $\gamma^0 = \sigma^2$, $\gamma^1 = i\sigma^1$, $C = -\gamma^0$, $\gamma^5 = \gamma^0\gamma^1 = \sigma^3$ (so $\gamma^5\gamma^5 = \mathbb{1}$). The Majorana spinors are then real: $\psi = -\gamma^0\bar{\psi}^T = \psi^*$. The Grassmann parameters in superspace are also Majorana spinors.

A supertranslation in superspace is $x_\mu \to x_\mu + i\bar{\varepsilon}\gamma_\mu\theta$, $\theta \to \theta + \varepsilon$. The generator for it is $Q = \partial/\partial\bar{\theta} + i\not{\partial}\theta$, and $\{Q, \bar{Q}\} = -2i\not{\partial}$. The corresponding supercovariant derivative is $D = \partial/\partial\bar{\theta} - i\not{\partial}\theta$, and it anticommutes with Q.

Grassmann integration is equivalent to differentiation:

$$\int d\theta^a = 0,\ \int d\bar{\theta}^a\ \theta^b = \delta^{ab} = \frac{\partial}{\partial\theta^a}\ \theta^b$$

Upon contraction of spinor indices, this normalizes the

Grassmann measure:

$$\int \frac{d^2\theta}{2}\,\bar{\theta}\theta = \int d\bar{\theta}\cdot d\theta\,\frac{\bar{\theta}\cdot\theta}{2} = 1\ .$$

The equal time anticommutation for Majorana spinors is:

$$\{\psi(x^1),\ \psi(y^1)\} = \delta(x^1-y^1)\ .$$

In two dimensions, the plane wave expansion for these real, two component spinors is:

$$\psi(x) = \frac{1}{\sqrt{4\pi}}\int_{-\infty}^{\infty} dp^1[a(p^1)e^{-ip\cdot x} + a^\dagger(p^1)e^{ip\cdot x}]u(p^1)$$

where $\{a(p),a^\dagger(k)\}=\delta(p-k)$ and $u(p^1)=\begin{pmatrix}\theta(p^1)\\ \theta(-p^1)\end{pmatrix}$, whence:

$$\bar{u}(p^1)u(p^1) = 0 \qquad \bar{u}(-p^1)u(p^1) = i(1-2\theta(-p^1))$$

$$\bar{u}(p^1)\gamma_5 u(p^1) = 0 \qquad \bar{u}(-p^1)\gamma_5 u(p^1) = i$$

$$\bar{u}(p^1)\gamma^\mu u(p^1) = 1 + 2\,g^{\mu 1}\theta(-p^1) \qquad \bar{u}(-p^1)\gamma_\mu u(p^1) = 0\ .$$

REFERENCES

[1] P. Fayet and J. Iliopoulos, Phys. Lett. 51B (1974) 461.

P. Fayet, Phys. Lett. 58B (1975) 67.

L. O'Raifeartaigh, Nucl. Phys. B96 (1975) 331.

A. Slavnov, Nucl. Phys. B124 (1977) 301.

E. Egorian and A. Slavnov, Ann. Phys. 116 (1978) 358.

[2] A. Salam and J. Strathdee, Phys. Lett. 49B (1974) 465.

A. Salam and J. Strathdee, Lett. Math. Phys. 1 (1975) 3. For a review, see:

A. Salam and J. Strathdee, Fort. der Physik 26 (1978) 57.

[3] W. Bardeen (1975), unpublished.

B. deWit and D. Freedman, Phys. Rev. Lett. 35 (1975) 827.

B. deWit and D. Freedman, Phys. Rev. D12 (1975) 2286.

[4] E. Witten, Nucl. Phys. B185 (1981) 513.

E. Witten, Phys. Lett. 105B (1981) 267.

[5] D. Volkov and V. Akulov, Phys. Lett. 46B (1973) 109.

[6] D. Volkov and V. Soroka, Jetp. Lett. 18 (1973) 312.

A. Pashnev, Theor. and Math. Phys. 20 (1974) 725.

B. Zumino, Nucl. Phys. B127 (1977) 189 (this makes contact with the language of Refs. [8]).

K. Shima, Phys. Rev. D15 (1977) 2165.

J. Schonfeld (1977) unpublished.

[7] E. Ivanov and A. Kapustnikov, J. Phys. A11 (1978) 2375.

[8] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2239; C. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2247.

A. Salam and J. Strathdee, Phys. Rev. 184 (1969) 1750.

[9] For reviews see: S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41 (1969) 531.

B. Lee, Chiral Dynamics, Gordon and Breach, New York, 1972, and references therein. A partial list (that we have especially profited from) includes:

S. Weinberg, Phys. Rev. Lett. 18 (1967) 507.

J. Schwinger, Phys. Lett. 24B (1967) 473.

S. Weinberg, Phys. Rev. 166 (1968) 1568.

W. Bardeen and B. Lee, Phys. Rev. 177 (1968) 2389.

R. Dashen and M. Weinstein, Phys. Rev. 183 (1969) 1261.

[10] M. Gell-Mann and M. Lévy, Nuov. Cim. 16 (1960) 705.

[11] M. Roček, Phys. Rev. Lett. 41 (1978) 451.

[12] W. Bardeen and V. Višnjić, Fermilab-Pub-81/49-THY, June 1981, to appear in Nucl. Phys. B.

[13] S. Ferrara, Lett. Nuov. Cim. 13 (1975) 629.

P. DiVechia and S. Ferrara, Nucl. Phys. B130 (1977) 93.

A. D'Adda, P. diVechia, and M. Lüscher, Nucl. Phys. B152 (1979) 125.

[14] L. Alvarez-Gaumé, D. Freedman, and M. Grisaru, Harvard preprint HUTMP 81/B111.

[15] S. Ferrara and B. Zumino, Nucl. Phys. B87 (1975) 207.

[16] e.g. see L. Brink, J. Schwarz and J. Scherk, Nucl. Phys. B121 (1977) 77, and references therein.

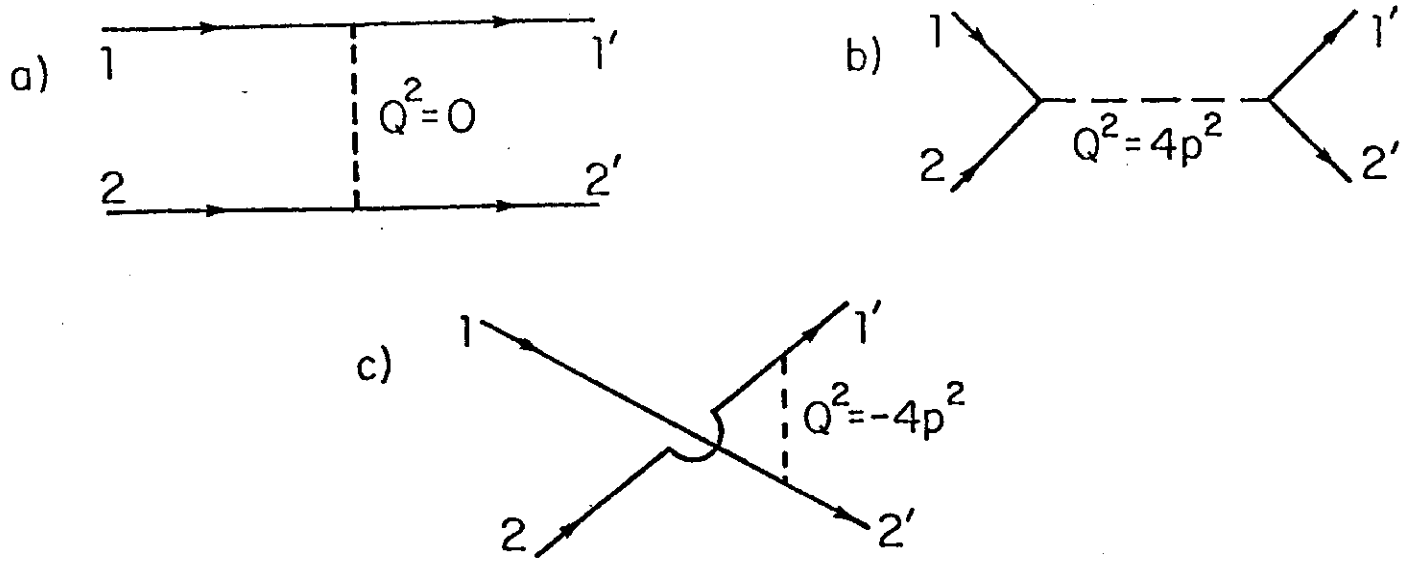

Fig. 1: Boson exhange in fermion-fermion scattering for the linear and nonlinear cases. The t-channel (a) is zero in both cases. The s-(b) and u-channels (c) are nonzero and interfere with a minus sign due to fermi statistics.

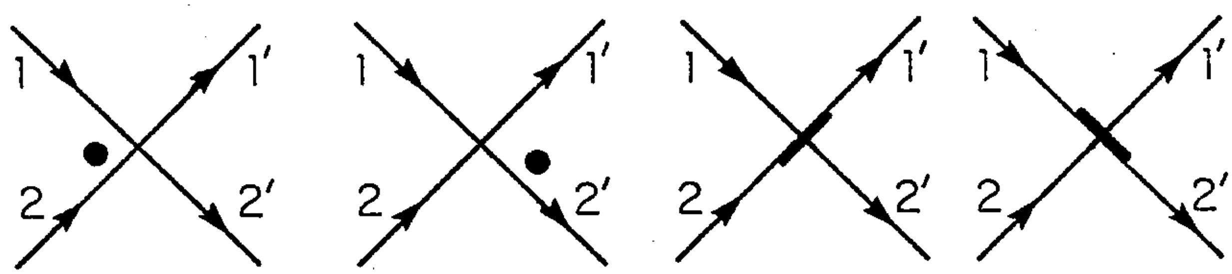

Fig. 2 Four-fermi contact interaction diagrams for the nonlinear case (4.1). The dot or heavy line denotes contraction of the derivative couplings on the adjacent lines. All four diagrams have equal magnitudes and interfere constructively. The two omitted diagrams that would have a dot in the remaining positions vanish by antisymmetry.

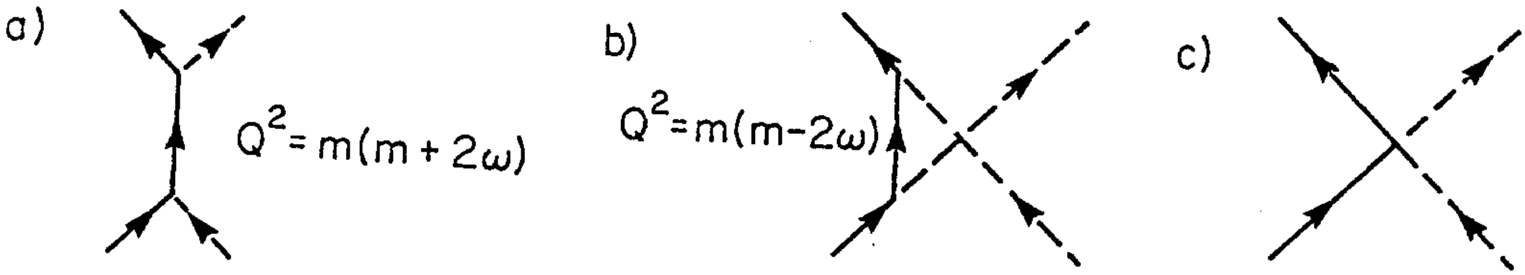

Fig. 3 Forward fermion-scalar tree amplitudes. Only (a) and (b) contribute in the linear theory, whereas all three do in the nonlinear one.

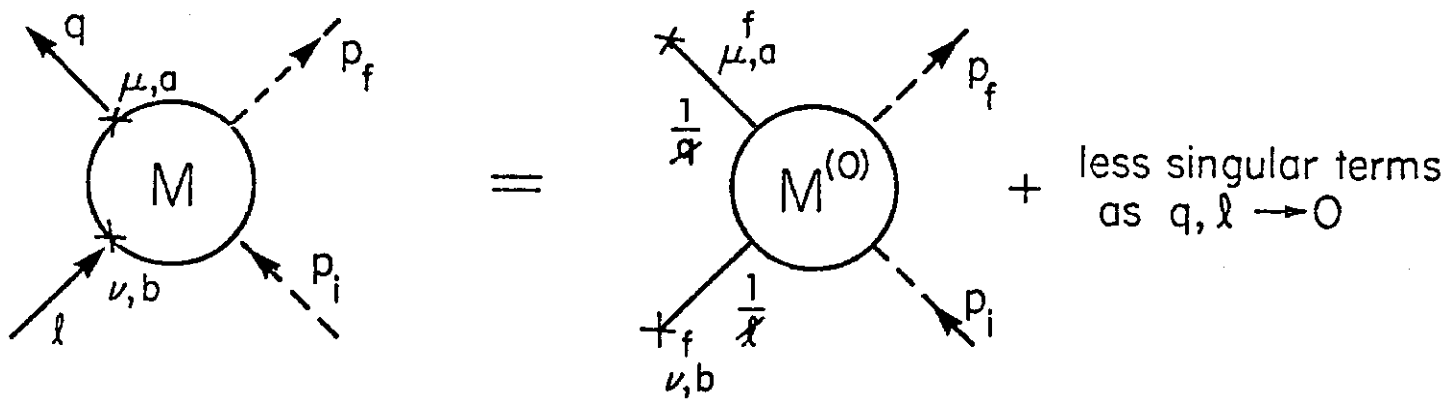

Fig. 4 Matrix element for a scalar state interacting with two supercurrents.